Solving mathematical problems: suggestions for AI from a study on primary school children

**Theoretical background**
Cognitive science and artificial intelligence (AI) both contribute to the understanding of intelligent behavior, although each one draws from a different perspective. From the point of view of cognitive science, intelligent behaviour can be defined as the “ability to acquire, understand and use knowledge” in order to achieve specific goals (Passer & Smith, 2004). Cognitive abilities that underlie these processes are numerous, as described by the Cattell-Horn-Carroll theory (CHC; Schneider & McGrew, 2012), which is one of the better-known psychometric-based taxonomies. This theory is an integration of two previously established theoretical models of intelligence: the theory of fluid and crystallised intelligence (Cattell, 1963; Horn et al., 1981) and the Carroll’s three-stratum model of intelligence (Carroll, 1993). CHC identifies five broad cognitive abilities (i.e., inductive and deductive reasoning, memory, cognitive speed, acquired knowledge, sensory and motor-linked abilities) that constitute general intelligence.

From the AI point of view, several Cognitive Architectures (e.g., ACT-R, Anderson, 1983; SOAR, Newell, 1990) have been proposed and implemented in the past in order to emulate intelligent behavior in problem-solving. In the last twenty years, AI has been specialized in specific sub-fields implementing computer programs able to perform a range of tasks (e.g., playing the game of chess), to interpret and respond to speech, to analyse diagrams, and so on. However, to our knowledge software, able to autonomously solve mathematical problems, do not exist. Indeed there are specialized programs, such as “Constraint Solvers” (Apt, 2003), which are able to solve mathematical puzzles but only with the human contribution in order to understand the text in natural language, extrapolate the relevant data and choose the best algorithm for the solution (Chesani et al., 2017).

Cognitive science can make a contribution to AI in this field, analysing how people deal with mathematical problem-solving (see Jonassen, 2000), adopting successful (and unsuccessful) strategies. Although several studies have identified skills (i.e., reading and calculation) and cognitive processes (i.e., semantic processing, working memory, long-term and short-term memories, inductive and deductive reasoning) involved in solving basic “word-mathematical problems” by primary school children (e.g., Swanson & Beebe-Frankenberger, 2004), the literature about the understanding of problem-solving strategies followed by children is less abundant. Thus, the analysis of these strategies undertaken by primary school children could provide useful information and insights for the development of AI programs able to autonomously solve word-mathematical problems.

**Aims and Hypotheses**
The main aim of this research project is to deepen the analysis of mathematical reasoning in primary school children. In particular, we will study problem-solving of children, in order to identify both the usual strategies used in solving problems of increasing complexity and the specific characteristics of the problems that make the resolution easier or harder. The problems will be featured by four characteristics: the presence/absence of mathematical operations (i.e., sum, division, subtraction and multiplication), the number of variables, constraints and domains. An example of a problem is the follow: *Mario, Filippo and Luca are fans of different football teams. Mario supports a team different from Filippo, who roots for another team compared to Luca. If the teams are Milan and Juve, which teams do the three friends support?* This problem presents three variables (Mario, Filippo, Luca), two domains (Milan, Juve) and two constraints (Mario≠Filippo; Filippo≠Luca).
In summary, two research questions will direct the present research project:

1. Does increasing the complexity of the problems (in terms of number of constraints, variables and domains) influence children’s performance?
2. Do children use different strategies in solving mathematical problems depending on the complexity of the problems?

**Methods**

Various mathematical problems, created *ad hoc*, will be administered during school time, to 71 students (as suggested by the GPower 3.1 analysis; Faul et al., 2007) in the 4th year of primary school. We choose an age range of 9/10 because the children need to have a relatively fluent reading skills and basic mental calculation ability in order to solve the mathematical problems presented (see Swanson & Beebe-Frankenberger, 2004).

After obtaining informed consent from their parents, each child will be evaluated by some tests in order to ascertain whether their logical-deductive capacities (Colored Raven Matrices; Raven et al., 1998) and their mental arithmetic computation abilities (subtest of the WISC-IV; Wechsler, 2012) are in the norm. All the children will then be given individually mathematical problems. In order to highlight the strategies used, a semi-structured interview will be administered for each problem presented, in which children will have to indicate which elements of the problem they evaluate during problem-solving and what calculations they perform. Children will be engaged for about two hours. Data on children with logical-deductive capacities and mental arithmetic abilities in the norm will then be analyzed using SPSS 23.0 package.

The tutor agrees to request approval of the research project from the ethics committee of the University of Bologna.

**Expected results and Implications**

Firstly, we expect that the complexity of mathematical problems (in terms of constraints, variables, and domains) will influence performances: more complex the problems are, more errors children do. Secondly, we suppose that children differ in their problem-solving strategies when they make errors, whereas they use a unique successful strategy when they give the correct solution.

The identification of successful and unsuccessful problem-solving strategies used by children, can provide useful information for the implementation of fully-fledged “intelligent softwares” able to understand the relevant data of the problems presented in natural language, reach the available problem-solving algorithms and choose the best in order to solve them. These softwares could also support children with math difficulties in solving problems, thus stimulating their interest in mathematics.
Plan of activities
The project activities will be concerned with scientific and methodological aspects. As regards the former, in the first two months of the research project the research fellow will perform an accurate literature review about the following topics:
- cognitive processes and skills involved in problem-solving of mathematical puzzles in primary school children;
- collecting and analysing a sufficient number of mathematical problems in order to create a corpus of examples useful for research activities.
As regards the latter aspect, starting from the third month of the research project the research fellow will carry out the following activities:
- individuation of skills and strategies necessary to solve mathematical puzzles by “intelligent software” (e.g., Constraint Solvers) and human agents, such as primary school children (3rd month);
- starting from mathematical problems collected from literature, creation of the “word mathematical problems” of different complexity (e.g., various number of constraints, variables and domains) that will be used for the study (4-5 th month);
- enrolment of participants and data analyses (6-9 th month);
- discussion of the data obtained and preparation of a paper (10-12 th month).